Correspondence

Regenerative and Pull-In Modes of Millimeter-Wave Reflex Klystron Amplifier

A reflex klystron used as an amplifier can operate in both the regenerative[1] and pull-in mode.[2] From articles published regarding reflex klystron amplification,[3-5] it appears that considerable confusion exists on this subject. It is the purpose of this communication to distinguish clearly between the two phenomena. The most readily noticeable differences occur in the frequency behavior and the input-output characteristics.

The significant difference in behavior of the two modes is clearly shown by Figs. 1 and 2. If the signal is shut off, there is no output from the tube operated in the regenerative mode; however, if the tube is operated in the pull-in mode, the tube output, pulled in with sufficient input existing, will pull away from the signal frequency to its free running frequency of oscillation, as far away as 200 Mc. Now, if the pass band of the receiver is limited, this oscillation may fall outside the frequency range covered by Figs. 1 and 2 and the two modes would appear indistinguishable from this aspect. To ascertain pull-in operation, the oscillation could then be located by tuning the receiver.

The two modes can also be distinguished by their input-output characteristics, as seen from Figs. 3 and 4. While the input-output characteristics in the regenerative mode are relatively smooth (Fig. 3), the input-output characteristics for the pull-in mode can be broken into three distinct sections, namely, free running, pull-in transition, and complete pull-in sections, as seen from Fig. 4.

In general, it was found experimentally that the gain (for comparable input levels), minimum detectable signal, and noise figure were larger for the pull-in mode than for the regenerative mode. Though as yet not conclusive, this also appears to apply for bandwidth. For example, noise figure of the pull-in mode was found to be greater than 40 db while for the regenerative mode it was less than 30 db.

Thus, by gradually varying the input level or removing the input signal altogether, the behavior of the amplifier output indicates the mode of operation.

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[1] K. Ishii "X-band receiving amplifier," *Electronics*, vol. 28, pp. 202–210; April, 1955.

[2] R. C. Mackey, "Injection locking of klystron oscillators," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 228–235; July, 1962.

[3] C. F. Quate, R. Kompfner, and D. A. Chisholm, "The reflex klystron as a negative resistance type amplifier," IRE TRANS. ON ELECTRON DEVICES, vol. ED-5, pp. 173–179; July, 1958.

[4] I. Thomas and L. Bounds, "The Investigation of the Characteristics of the KS 9-20A Reflex Klystron When Used as an Amplifier," The Mullard Radio Valve Co., Ltd., Waddon, England, D.V.T. Rept. No. U231; 1961.

[5] T. Matsumoto, M. Suzuki, and C. Funatsu, "Noise characteristics of reflex klystron amplifier," *J. Inst. Elec. Commun. Engrgs. of Japan*, vol. 46, pp. 1225–1229; September, 1963.

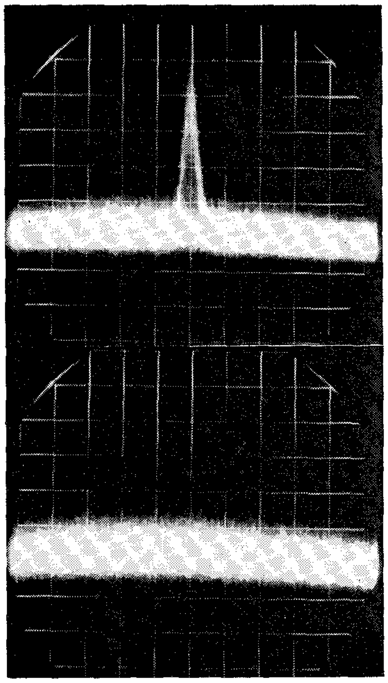

Fig. 1—Regenerative mode of reflex klystron amplifier VA-250. Upper picture shows amplified signal, gain at 70.19 kMc is 20 db, $V_a=1587$ v, $V_r=-446$ v. Lower picture shows amplifier output with no signal input. Horizontal axis is frequency swept. Input signal is 1000 cps square wave modulated and generated by a VA99 reflex klystron.

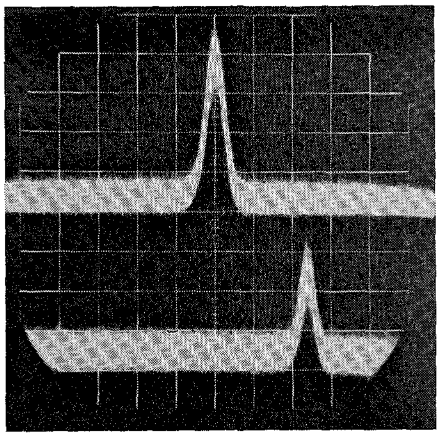

Fig. 2—Pull-in mode of reflex klystron amplifier VA-250. Upper picture shows oscillator locked into the CW signal frequency of 70.13 kMc. Gain is 10 db. Lower picture shows amplifier is now a free running oscillator at 70.17 kMc with input removed. Horizontal axis is frequency swept. $V_a=1460$ v and $V_r=-222$ v.

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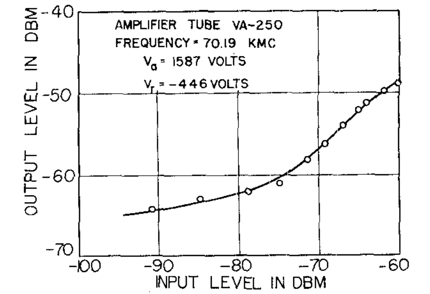

Fig. 3—Input-output characteristics of VA-250 reflex klystron amplifier operated in regenerative mode.

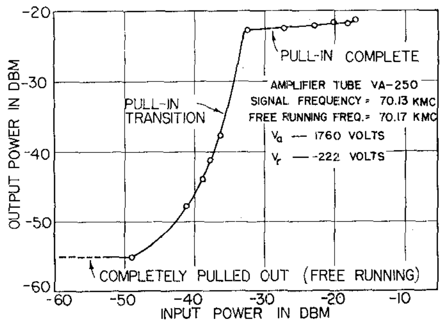

Fig. 4—Input-output characteristics of VA-250 reflex klystron amplifier operated in pull-in mode.

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An Additional Relation for "Design of Mode Transducers"

Solymar and Eaglesfield,[1] and recently Wolfert,[2] have disclosed analytical methods for designing the contours of tapered waveguide mode transducers to favor the desired modes and suppress the spurious modes. Basically, the methods are similar, but they differ in the selection of the mode weighting functions and in the methods of plotting the cross-sectional boundary contours. The

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[1] L. Solymar and C. C. Eaglesfield, "Design of mode transducers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 61–65; January, 1960.

[2] P. H. Wolfert, "A wide-band rectangular-to-circular mode transducer for millimeter waves," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (*Correspondence*), vol. MTT-11, pp. 430–431; September, 1963.

transducers designed by these methods probably have very high efficiency-to-length ratios, but mathematical proof of this still is lacking.

It is the purpose of this communication to suggest that the weighting functions are not as arbitrary as they first appear. An additional relationship is imposed by the Kinetic Power Theorem.[3]

In the notation of Solymar and Eaglesfield,[1]

$$\psi_m(x, y, z) = g_1(z)\psi_m^A + g_2(z)\psi_m^B \quad (1)$$

is a gradually-varying eigenfunction that characterizes signal propagation on the mth transmission line of the tapered transducer. Weighting functions $g_1(z)$ and $g_2(z)$ cause ψ_m to vary from a value of ψ_m^A (the eigenfunction of the input mode) at $z=0$, to a value of ψ_m^B (the eigenfunction of the output mode) at $z=L$. When ψ_m is constructed in this way, a waveguide cross-sectional boundary can be derived from the orthogonal trajectories of the equipotential contours, as by Solymar and Eaglesfield,[1] or from the normals to the E-field vectors, as by Wolfert.[2] Since the E-field vectors are proportional to the gradients of ψ_m, the two derivations yield the same results. The problem is to find the functions $g_1(z)$ and $g_2(z)$ that optimize the design. This is an extremely complicated mathematical problem for which only empirical approaches have been suggested thus far.

To put the problem in perspective, one may write the following equations relating ψ_m, the scaler magnitude of the Hertzian vector potential, and the field components of a TE-mode propagating on the mth transmission line.

$$H_{tm} = j\beta_m \nabla_t \psi_m e^{-j\beta_m z} \quad (2)$$

$$E_{tm} = j\omega\mu_0 k \times H_{tm} \quad (3)$$

$$H_{zm} = k_{cm}^2 \psi_m e^{-j\beta_m z} \quad (4)$$

where β_m and k_{cm} are the propagation and cutoff constants, respectively, of the line, and k is a unit vector on the z axis. It is assumed that β_m and k_{cm} are either constant, or vary sufficiently slowly with z, that hybrid modes are not excited. Actually, this assumption is implied by the separation of variables in (1).

A complex Poynting vector S_m for the mth line can be formed from (2) and (3):

$$S_m = E_{tm} \times H_{tm}^* = \omega\mu_0\beta_m(k \times \nabla_t\psi_m) \times \nabla_t\psi_m. \quad (5)$$

Expanding, and integrating over a waveguide cross section $S(z)$, gives the power flowing in the line

$$P_m = \frac{\omega\zeta\beta_m k_{cm}^2}{2c}\int_{S(z)} \psi_m^2 da \quad (6)$$

where $\zeta = 377\ \Omega$, and c is the velocity of light. If mks units are used, ψ_m is expressed in ampere-meters to yield P in watts.

Although β_m and k_{cm} can be assumed to be invariant from one end of the transducer to the other ($\beta_m^A = \beta_m^B$, $k_{cm}^A = k_{cm}^B$) in general, by the nature of its construction, the bounded area S will be a slowly-varying function of z. Nevertheless, for this lossless, reflectionless transducer, the Kinetic Power Theorem requires that

$$\frac{2c}{\omega\zeta\beta_m k_{cm}^2}\frac{dP}{dz} = 0 = \frac{d}{dz}\int_{S(z)} [g_1(z)\psi_m^A + g_2(z)\psi_m^B]^2 da. \quad (7)$$

Eq. 7 provides a relationship between $g_1(z)$ and $g_2(z)$ that depends on the behavior of the boundary. Unfortunately, the equation cannot be integrated without first solving the boundary-value problem, but when the boundary has been derived by the graphical methods discussed in the references, a numerical integration can be carried out to test the validity of the functions. In view of the slowly-varying nature of the cross section, and the fact that $g_1(0)$ and $g_2(L)$ are readily adjusted to equalize the powers in waveguides A and B, it probably is sufficient to carry out the integration at one intermediate cross section, such as at $z=L/2$.

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[3] The Kinetic Power Theorem states that any loss in real kinetic power must equal the power delivered externally. It implies a conservation of power flow through a lossless, reflectionless transducer.

On the Focused Fabry-Perot Resonator in Plasma Diagnostics

In a recent paper, Primich and Hayami[1] discussed the application of a Fabry-Perot resonator to plasma diagnostics. In particular, they studied plasma densities that are uniform in space and that have a plasma frequency much less than the operating frequency of the microwave system. They also restrict themselves to plasmas that are collisionless.

The purpose of this communication is to study the use of a Fabry-Perot resonator in diagnostics of a low density but nonuniform plasma and also to consider the plasma to have collisions. Such a problem would arise in determining the plasma parameters in the wake of a projectile or in various low density plasma confinement devices.

Consider the resonator shown in Fig. 1 with a plasma slab inserted in it. One can consider plane waves[2,3] to be propagating in the resonator, hence the plasma slab may be considered as a dielectric medium of dielectric constant ϵ_{p1}.

$$\epsilon_{p1} = \epsilon_0\left[1 - \frac{\omega_p^2}{\omega^2 - j\omega\nu}\right] \quad (1)$$

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[1] R. I. Primich and R. A. Hayami, "The application of the focused Fabry-Perot resonator to plasma diagnostics," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-12, pp. 33–42; January, 1964.
[2] G. Goubau and F. Schwering, "On the guided propagation of electromagnetic wave beams," IRE Trans. on Antennas and Propagation, vol. AP-9, pp. 248–256; May, 1961.
[3] W. Culshaw, "Further considerations on Fabry-Perot type resonators," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 331–339, September, 1962.

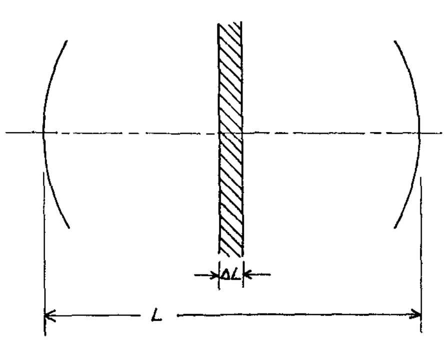

Fig. 1—Fabry-Perot plasma diagnostic device.

where $\omega_p^2 = ne^2/m\epsilon_0$ is the plasma frequency and ν is the collision frequency. For $\nu \ll \omega$, (1) becomes

$$\epsilon_{p1} \approx \epsilon_0\left[1 - \frac{\omega_p^2}{\omega^2} + j\frac{\omega_p^2\nu}{\omega^3}\right]. \quad (2)$$

Consider now the Fabry-Perot cavity to be resonated in the absence of any plasma medium. Finite losses are present resulting from the reflecting end plates and diffraction, hence the complex resonant frequency is expressible as

$$\Omega_0 = \omega_0\left[1 + \frac{j}{2Q_0}\right] \quad (3)$$

where ω_0 is the natural resonant frequency and Q_0 is the Q of the empty cavity. Introduction of the plasma slab will cause the resonant frequency to shift by an amount $\delta\omega$; hence the new complex resonant frequency may be written as

$$\Omega_1 = (\omega_0 + \delta\omega)\left[1 + \frac{j}{2Q_1}\right]. \quad (4)$$

As the Q's of the Fabry-Perot cavity are very high, one obtains

$$\frac{\Omega_1 - \Omega_0}{\omega_0} \approx \frac{\delta\omega}{\omega_0} + \frac{j}{2}\left[\frac{1}{Q_1} - \frac{1}{Q_0}\right]. \quad (5)$$

The fields before and after the introduction of the plasma slab will be relatively unchanged as $\epsilon_{p1} \approx \epsilon_0$, hence the use of the perturbational concept[4] is valid which yields

$$\frac{\Omega_1 - \Omega_0}{\Omega_0} = \frac{-\iiint_{\Delta V}(|\Delta\epsilon|E_0|^2 + \Delta\mu|H_0|^2)dV}{\iiint_V(\epsilon_0|E_0|^2 + \mu_0|H_0|^2)dV} \quad (6)$$

where in general Ω, μ, and ϵ are complex. At resonance, the electric and magnetic energies are equal and the plasma does not alter the permeability, hence (6), using (2) and (5) becomes

$$\frac{\Omega_1 - \Omega_0}{\omega_0} = \frac{\iiint_{\Delta V}\left[\frac{\omega_p^2}{\omega_0^2} - j\frac{\omega_p^2\nu}{\omega_0^3}\right]|E_0|^2 dV}{2\iiint |E_0|^2 dV} = \frac{\delta\omega}{\omega_0} + \frac{j}{2}\left[\frac{1}{Q_1} - \frac{1}{Q_0}\right]. \quad (7)$$

[4] R. F. Harrington, "Time-Harmonic Electromagnetic Fields," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 317–380; 1961.